A NEW SELP–INITIATED WLS APPROXIMATION METHOD FOR THE DESIGN OF TWO–DIMENSIONAL EQUIRIPPLE FIR DIGITAL FILTERS

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Abstract:
In this paper, motivated by Chi and Chiou's weighted least–squares (WLS) Chebyshev approximation method [5] which is for the design of 1–D (one–dimensional) FIR digital filters with arbitrary complex frequency response, we propose a new self–initiated iterative WLS approximation method for the design of 2–D equiripple FIR digital filters with arbitrary 2–D complex frequency response. Several circularly symmetric lowpass filter design examples are provided to justify the good performance of the proposed approximation method.

1. Introduction

Various two–dimensional digital filter design methods have been reported in the open literature such as the windowing method, the frequency transformation method and the approximation method. The approximation method usually needs more computational efforts compared with other methods, whereas, it is still most useful because it is free from windowing effects; it can accommodate different approximation errors in different frequency bands; the required order is usually smaller compared with other methods for the same specifications. In particular, minimax (Chebyshev) criterion based filters can only be obtained by approximation methods such as exchange ascent algorithms [1,2], and Algazi, Suk and Rim's WLS method [3] which is basically an extension of 1–D Lawson's algorithm.

Recently, Chi and Kou [4] proposed a new efficient WLS Chebyshev approximation method for the design of 1–D equiripple linear–phase FIR filters. Based on Chi and Kou's method, Chi and Chiou [5] proposed another WLS Chebyshev approximation method which is computationally more efficient than Chi and Kou's method and can approximate any 1–D arbitrary complex frequency response. In this paper, motivated by Chi and Chiou's WLS Chebyshev approximation method, we propose a new WLS approximation method for the design of 2–D equiripple FIR digital filters. The proposed method can also approximate any arbitrary two–dimensional complex frequency response.

2. The New 2–D WLS Approximation Method

The frequency response of the 2–D FIR digital filter with a finite domain of support S to be designed can be expressed by

$$H(\omega_1,\omega_2) = \sum_{(n_1,n_2)\in S}\sum h(n_1,n_2)\, e^{-j(\omega_1 n_1+\omega_2 n_2)}. \quad (1)$$

The approximation error $E(\omega_1,\omega_2)$ is defined as

$$E(\omega_1,\omega_2) = H_d(\omega_1,\omega_2) - H(\omega_1,\omega_2) \quad (2)$$

where $H_d(\omega_1,\omega_2)$ is the desired frequency response. Assume that $H_d(\omega_1,\omega_2)$ is defined over p disjoint nontransition bands $\mathcal{R}_1, \mathcal{R}_2, ..., \mathcal{R}_p$, and that the desired tolerance error ratio among $\mathcal{R}_1, \mathcal{R}_2, ..., \mathcal{R}_p$, is $\rho_1:\rho_2:\cdots:\rho_p$ with $\min\{\rho_1,\rho_2, ..., \rho_p\}=1$. Our objective is to find a set of filter coefficients $\{h(n_1,n_2)\,|\,(n_1,n_2)\in S\}$ such that $H(\omega_1,\omega_2)$ is of equiripple with the resultant approximation error ratio $\delta_1:\delta_2:\cdots:\delta_p = \rho_1:\rho_2:\cdots:\rho_p$ where $\delta_k = \max\{|E(\omega_1,\omega_2)|, (\omega_1,\omega_2)\in\mathcal{R}_k\}$ is the maximum approximation error in $\mathcal{R}_k$. Next, let us present the WLS estimator on which the new approximation method is based.

For simplicity, assume that the filter to be designed is zero–phase with real filter coefficients. Under this assumption, the frequency response $H(\omega_1,\omega_2)$ is real due to $h(n_1,n_2) = h(-n_1,-n_2)$ and can be expressed as

$$H(\omega_1,\omega_2) = h(0,0)+ \sum_{(n_1,n_2)\in S'}\sum 2\, h(n_1,n_2) \cdot \cos(\omega_1 n_1+\omega_2 n_2) \quad (3)$$

where S' is a set such that S is equal to the union of three mutually exclusive sets, $\{(0,0)\}$, S' and S'' where S'' is S' flipped with respect to the origin. Let us consider M sampling points in the set

$$\mathcal{R} = \mathcal{R}_1 \cup \mathcal{R}_2 \cup \cdots \cup \mathcal{R}_p,$$

denoted $(\omega_{1i},\omega_{2i})$, i=1,2, ..., M, at which we express the approximation errors in the following vector form:

$$\underline{E} = \underline{H}_d - D\cdot\underline{h} \quad (4)$$

where

$$\underline{E} = (E(\omega_{11},\omega_{21}), ..., E(\omega_{1M},\omega_{2M}))^t,$$

$$\underline{H}_d = (H_d(\omega_{11},\omega_{21}), ..., H_d(\omega_{1M},\omega_{2M}))^t,$$

$\underline{h}$ is a vector containing all nonredundant $h(n_1,n_2)$'s and D is a matrix with proper dimension whose components can be easily determined from (3). It is well–known that the WLS estimate, $\hat{\underline{h}}$, is given by [6]

$$\hat{\underline{h}} = [D^t\, W\, D]^{-1}\, D^t\, W\, \underline{H}_d \quad (5)$$

where $W=\text{diag}[w_1,w_2, ... ,w_M]$ with $w_i\geq 0$, which minimizes the sum of weighted error squares

$$J(\underline{h}) = \underline{E}^t\, W\, \underline{E} \quad (6)$$

and possesses a well–known property as follows:

(P1) *The larger the weight* w_i, *the smaller is the absolute error* $|E(\omega_{1i},\omega_{2i})|$.

Let us define some notations for latter use:

- Piecewise–constant function $W_e(\omega_1,\omega_2)$, $(\omega_1,\omega_2)\in\mathcal{R}$:

$$W_e(\omega_1,\omega_2) = 1/\rho_k, \text{ if } (\omega_1,\omega_2)\in\mathcal{R}_k \quad (7)$$

- Error ripple $e_k^j(\omega_1,\omega_2)$ in $\mathcal{R}_k$:

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$$e_k^j(\omega_1,\omega_2) = E(\omega_1,\omega_2),\ (\omega_1,\omega_2)\in \mathcal{R}_k^j \subset \mathcal{R}_k \quad (8)$$

where either $e_k^j(\omega_1,\omega_2)\geq 0$ or $e_k^j(\omega_1,\omega_2)\leq 0$ for all $(\omega_1,\omega_2)\in\mathcal{R}_k^j$, and $\mathcal{R}_k^j$ is a simply connected region such that $\mathcal{R}_k = \mathcal{R}_k^1 \cup \mathcal{R}_k^2 \cup \cdots \mathcal{R}_k^m$, where m is the total number of error ripples in $\mathcal{R}_k$.

- Weighted error ripple amplitude $a_{k,j}$:

$$a_{k,j}=\max\{|W_e(\omega_{1i},\omega_{2i})\cdot e_k^j(\omega_{1i},\omega_{2i})|,(\omega_{1i},\omega_{2i})\in\mathcal{R}_k^j\} \quad (9)$$

The new WLS approximation method, which is shown in Figure 1, is an iterative algorithm based on **(P1)** for finding the optimum weight w_i such that $|E(\omega_1,\omega_2)|$ is of equiripple with the desired approximation error ratio among the nontransition bands. It begins with the initial weighting function

$$w_i^{(0)} = W_e(\omega_{1i},\omega_{2i}),\ (\omega_{1i},\omega_{2i})\in\mathcal{R},\ i=1,2,\ldots,M. \quad (10)$$

Assume that we ended up with the weighting function $w_i=w_i^{(n-1)}$ at the (n−1)th iteration. For the nth iteration, the WLS estimate $\hat{\underline{h}}$ is computed by (5) with $w_i=w_i^{(n-1)}$. Then, the associated $\underline{E}$ is computed by (4) with $\underline{h}=\hat{\underline{h}}$. Next, we search for the weighted error ripple amplitudes $a_{k,j}$ for all k and j. Finally, we check whether the frequency response of the designed filter is of equiripple for $(\omega_{1i},\omega_{2i})\in\mathcal{R}$ by

$$(a_{max}-a_{min})/a_{max} \leq \alpha \quad (11)$$

where

$$a_{max}=\max_{k,j}\{a_{k,j}\},\quad a_{min}=\min_{k,j}\{a_{k,j}\},$$

and α is a preassigned small positive constant. If it is not of equiripple and n<L where L is the maximum allowed iteration number, we update the weighting function as follows:

$$w_i^{(n)}=\frac{w_i^{(n-1)}|W_e(\omega_{1i},\omega_{2i})\cdot E(\omega_{1i},\omega_{2i})|^q}{\max\{w_i^{(n-1)}|W_e(\omega_{1i},\omega_{2i})\cdot E(\omega_{1i},\omega_{2i})|^q\}} \quad (12)$$

where q is a positive real number and must be set in advance. Note that $0<w_i^{(n)}\leq 1$ for all $(\omega_{1i},\omega_{2i})\in\mathcal{R}$. Empirically, we found that the proposed method converges for $1\leq q<2$ and that q=1.5 is a good choice.

Although the proposed algorithm was illuminated via a zero–phase case for which $H(\omega_1,\omega_2)$ is real, it is applicable for any other case. For instance, the linear phase 2–D FIR filter design where $S = \{(n_1,n_2)|0\leq n_1\leq N_1,\ 0\leq n_2\leq N_2\}$ and $H_d(\omega_1,\omega_2) = A(\omega_1,\omega_2)\cdot\exp\{-j(\omega_1N_1+\omega_2N_2)/2\}$ can be performed by redefining $E(\omega_1,\omega_2)$ as $E(\omega_1,\omega_2) = \{H_d(\omega_1,\omega_2)-H(\omega_1,\omega_2)\}\cdot\exp\{j(\omega_1N_1+\omega_2N_2)/2\}$ which can be expressed as a real linear vector model given by (4). On the other hand, if $H_d(\omega_1,\omega_2)$ is an arbitrary complex frequency response with real filter coefficients, the error vector $\underline{E}$ given by (4) is complex. The WLS estimate $\hat{\underline{h}}$ can be similarly obtained. The reader can refer to [5] for details.

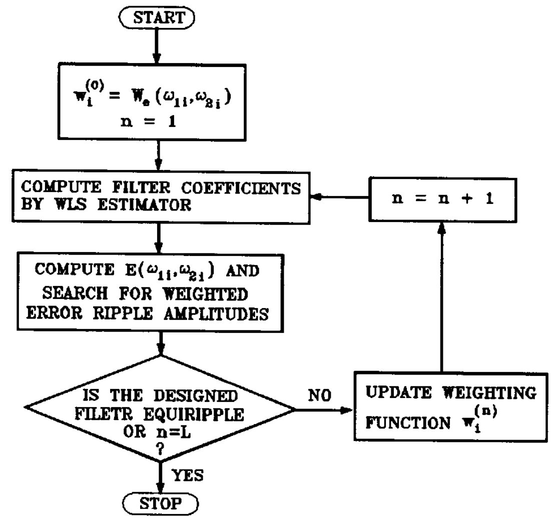

Fig. 1. The proposed WLS approximation method.

3. Design Examples

Let us show two lowpass filter design examples to justify the good performance of the proposed approximation method. Assume that the desired frequency response is zero–phase and circularly symmetric as follows:

$$H_d(\omega_1,\omega_2) = \begin{cases} 1, & (\omega_1,\omega_2)\in\mathcal{R}_1 \\ 0, & (\omega_1,\omega_2)\in\mathcal{R}_2 \end{cases} \quad (13)$$

where the passband $\mathcal{R}_1$ is the interior of the circle of radius $r_1=0.4\pi$ and the stopband $\mathcal{R}_2$ is the exterior of the circle of radius $r_2=0.6\pi$. The p=2 nontransition bands $\mathcal{R}_1$, $\mathcal{R}_2$ and the transition band are shown in Figure 2. Further assume that the frequency response $H(\omega_1,\omega_2)$ is eightfold symmetric. For this case,

$$h(n_1,n_2)=h(-n_1,n_2)=h(n_1,-n_2)=h(n_2,n_1). \quad (14)$$

Thus it is sufficient to approximate the desired frequency response in the shaded regions R_1 and R_2 indicated in Figure 2. Moreover, assume that the domain of support S $= \{(n_1,n_2)|-N\leq n_1\leq N,\ -N\leq n_2\leq N\}$, which implies that the FIR filter to be designed is an eightfold symmetric $(2N+1)\times(2N+1)$–point filter due to (14). Remark that the constraint given by (14) can be easily taken into account in formulating the linear vector model given by (4) so that the vector $\underline{h}$ only consists of nonredundant coefficients $\{h(n_1,n_2)|0\leq n_2\leq n_1\leq N\}$.

In the following two design examples, the sampling points $(\omega_{1i},\omega_{2i})$, i=1, 2, ..., M, include uniform sampling points on the Cartesian grid $(k_1\Delta\omega,\ k_2\Delta\omega)$ in the regions R_1 and R_2, $K_1 = \mathrm{Int}\{1+ r_1/(\sqrt{2}\cdot\Delta\omega)\}$ uniform sampling points on the boundary between R_1 and transition band, and $K_2 = \mathrm{Int}\{1+ r_2/(\sqrt{2}\cdot\Delta\omega)\}$ uniform sampling points on the boundary between R_2 and transition band, where Int[x] denotes the integer part of x. The sampling interval $\Delta\omega$ was assigned to $\Delta\omega=\pi/64$. After the optimum filter was obtained by the proposed approximation method with the convergence parameter α (see (11)) set to 0.01, we then searched for the maximum error via a very dense discrete Fourier transform of the optimum filter.

Example 1:

Assume that the desired tolerance error ratio between passband and stopband is equal to $\rho_1/\rho_2=1$. The parameter q (see (12)) was set to 1.5 in the proposed approximation method. The numerical results of the designed filters are shown in Table 1. For comparison, the corresponding results reported in [2] and [3] are also shown in Table 1. We can see, from Table 1, that maximum errors associated with the proposed method are slightly larger (around 1%) than the corresponding maximum errors associated with Harris' steepest ascent method [2], while they are smaller than those associated with the modified Lawson's method [3]. However, the modified Lawson's method spends fewer iterations.

On the other hand, we also performed the design of 9×9–point FIR filter using the proposed approximation method with different values of q. The maximum errors of the designed filters along with the total iterations spent are shown in Table 2. One can observe, from Table 2, that the proposed method is convergent for $1\leq q<2$ with the same maximum error. The parameter q plays a role similar to the step size parameter in the well–known least–mean–square (LMS) adaptive filter. By our experience, for a small q, it is overdamped in that $E(\omega_1,\omega_2)$ follows a smooth path with iteration number, and for a large q, it is underdamped in that $E(\omega_1,\omega_2)$ exhibits oscillations from iteration to iteration.

Example 2:

For this example, we employed the proposed approximation method with the parameter q set to 1.5 to design 9×9–point lowpass filters for different values of tolerance error ratio ρ_1/ρ_2 between passband and stopband. Frequency responses of the designed equiripple filters for $\rho_1/\rho_2 = 5$, 1, and 1/5 are shown in Figures 3a, 3b and 3c, respectively. The maximum errors δ_1 in passband and δ_2 in stopband of each designed filter as well as the ratio δ_1/δ_2 are shown in Table 3, from which, one can see that error ratios δ_1/δ_2 are very close to the corresponding values of ρ_1/ρ_2, and that δ_1 is larger and δ_2 is smaller for a larger ρ_1/ρ_2. Moreover, one can observe,

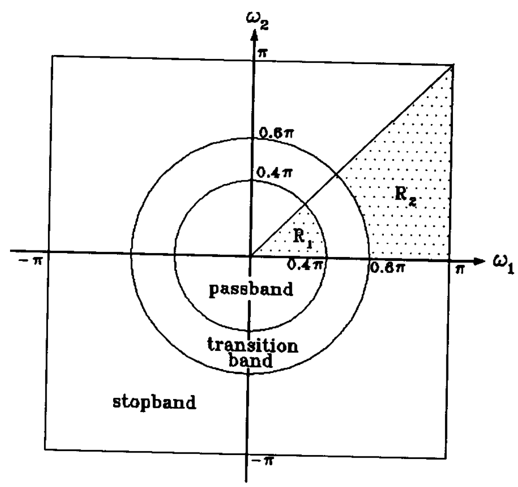

Fig. 2. Passband, stopband and transition band of the desired circularly symmetric lowpass filter.

Table 1.
Numerical results of the designed zero–phase eightfold symmetric lowpass filters associated with the proposed approximation method with q=1.5, and the corresponding results reported in [2] (Harris' steepest ascent method) and those reported in [3] (Modified Lawson's method). The tolerance error ratio (between passband and stopband) is $\rho_1/\rho_2=1$.

Order	Algorithm	Maximum error	Iteration number
7x7	The proposed method	0.1281	51
	Harris [9]	0.1272	—
	Lawson [11]	0.1336	9
9x9	The proposed method	0.1151	36
	Harris [9]	0.1141	—
	Lawson [11]	0.1202	8
11x11	The proposed method	0.0577	174
	Harris [9]	0.0569	—

Table 2.
Numerical results of the designed zero–phase eightfold symmetric 9×9–point lowpass filters by the proposed approximation method with different values of q. The tolerance error ratio (between passband and stopband) is $\rho_1/\rho_2=1$.

q	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
Iteration number	79	53	47	42	38	36	34	32	31	54	>500
Maximum error	0.1150	0.1149	0.1149	0.1149	0.1149	0.1149	0.1149	0.1148	0.1150	0.1149	-

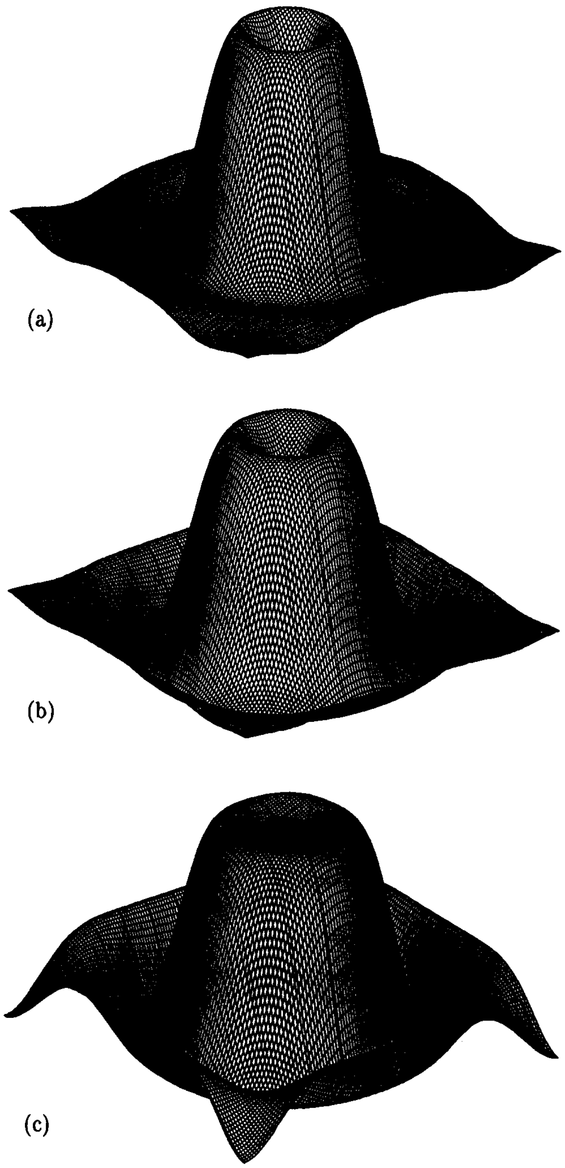

Fig. 3. Frequency responses of the designed zero–phase eightfold symmetric 9×9–point lowpass filters with tolerance error ratio (between passband and stopband) ρ_1/ρ_2 equal to (a) 5, (b) 1 and (c) 1/5, respectively.

from Figure 3, that for a larger ρ_1/ρ_2, the ripple in passband is deeper since δ_1 is larger, while the frequency response in stopband is flatter since δ_2 is smaller.

Table 3.
Maximum errors δ_1 (in passband) and δ_2 (in stopband) of the designed zero–phase eightfold symmetric 9×9–point lowpass filters with different values of tolerance error ratio ρ_1/ρ_2 (between passband and stopband).

ρ_1/ρ_2	δ_1	δ_2	δ_1/δ_2
5	0.1923	0.0386	4.981
1	0.1149	0.1148	1.000
1/5	0.0401	0.2014	0.199

4. Conclusions

We have presented a new self–initiated iterative WLS approximation method (see Figure 1) for the design of 2–D equiripple FIR filters which is basically an extension of Chi and Chiou's WLS approximation method [5] for the design of 1–D FIR digital filters with arbitrary complex frequency response. The proposed approximation method can be used to design real 2–D FIR filters with any arbitrary complex frequency response although we illuminated it via a zero–phase case, and the designed optimum filter basically is of equiripple. It allows the exact specification of the cutoff frequencies with no need of any initial guess for a suitable set of extremal frequencies or filter coefficients. We also showed two design examples of circularly symmetric lowpass filter which indicate that the performance of the proposed method is very close to the performance of exchange ascent algorithms [1,2]. Moreover, the proposed method does not have the degeneracy problem existing in exchange ascent algorithms because the WLS estimate given by (5) is associated with an overdetermined system of M>>dim{$\underline{h}$} linear equations of dim{$\underline{h}$} unknowns (see (4)). Besides, the WLS estimate can be efficiently computed in a recursive fashion [6].

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